MULTIPLE X-RAY TOMOGRAPHY USING TRANSMITTED, SCATTERED 
AND FLUORESCENT RADIATION

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ABSTRACT

A multiple CT-scanner is described, which contemporaneously uses transmitted, scattered and 
fluorescent X-rays for Imaging. The scanner is characterized by a small size X-ray tube and by four 
detectors: a “pencil” X-ray NaI(Tl) for transmitted tomography, a larger size NaI(Tl) for 90° 
Compton tomography, a thermoelectrically cooled Si-PIN or CdZnTe for fluorescent imaging and a 
CdZnTe for Rayleigh (or diffraction) tomography. Examples of applications are shown.

1. INTRODUCTION

X-Ray Transmission Computed Tomography (in the following CT) is a well established 
diagnostic technique in Medicine and Industry. It is non-destructive and allows to reconstruct the 
image of a cross-sections of a sample starting from a set of X-ray “radiographic” measurements 
taken at different angles. It is performed by placing a X-ray source and a X-ray detector aligned at 
opposite sides of the sample. The detector collects the part of the radiation emitted by the X-ray 
source that crosses the sample without interacting with it.

Interaction of monoenergetic incident radiation of energy $E_0$ with the sample is mainly 
depending on following effects (Figure 1):

a. photoelectric effect: the incident photon disappears and the related energy is transferred to an 
internal electrons of the atoms of the sample. Almost contemporaneously a fluorescent X-ray 
photon is emitted from the excited atom;
b. Compton effect: the incident photon transfers part of its energy to an external electron of an 
atom of the sample and is deviated from its originary trajectory;
c. Rayleigh effect: the incident photon is deviated by a bound electron from its originary trajectory 
without losing energy.

The attenuation of the radiation by the sample, supposed homogeneous, is due to the sum of the 
three above mentioned effects, and can be expressed as:

$$N_T = N_0 \exp(-\mu x)$$

where $N_0$ and $N_T$ are the radiation intensity emitted by the X-ray source and transmitted by the 
sample (and recorded by the detector) respectively, $\mu$ is the linear attenuation coefficient of the 
homogeneous sample of width $x$.

The reconstructed image represents a map of the cross section in terms of linear attenuation 
coefficient of the material, which is a function of the mean atomic number ($Z$) and/or density ($\delta$) in 
each single volume element (voxel).

When the photoelectric effect prevails, which occurs at low energies, then the linear attenuation 
coefficient is a function of both $Z$ and $\delta$, whereas in the Compton region it is a function of $\delta$ only. 
Rayleigh effect is only prevailing at very small angles, and is a function of $Z^2 - Z^3$ approximately, 
in the biological region.
CT is based on the registration of the incident photons **not interacting** with the sample material. However also photons may be considered which **interact** with the sample or are **produced in the sample**, and which also “contain” useful information about the sample itself. These are:

a. secondary X-ray photons due to photoelectric effect, which energy is characteristic of the atoms of the sample. These X-rays are emitted isotropically and are detected when emitted by elements with sufficiently high Z;

b. secondary X-ray photons due to Compton effect, which energy is not too different from energy of incident photons. The intensity of these photons is directly related to the density $\delta$ of the scattering volume. The emission of secondary Compton photons is almost isotropic;

c. X-ray photons due to Rayleigh effect, which energy is the same as that of incident photons. The intensity of these photons is related to a high power of the atomic number of the irradiating voxel, depending on incident energy and scattering angle. The emission of secondary Rayleigh photons is strongly forward peaked.

In this work an integrated tomographic system is presented, which is able to carry out contemporaneously transmission, scattering and fluorescence imaging. Advantages and disadvantages of each process will be discussed.

2. **THEORETICAL BACKGROUND**

Let us first consider a beam of collimated and monoenergetic photons of energy $E_0$ crossing a sample of thickness $x$.

2a. **transmission tomography**

If the sample is homogeneous Eq. (1) is valid, and the tomographic image is simply a constant density image. If the sample is inhomogeneous then it may be divided into a large number of voxels having a volume $(\Delta x)^3$, where $\Delta x$ has the same order of magnitude than the width of the incident beam. It is supposed that the attenuation coefficient remains constant into a single voxel.

In transmission tomography the X-or $\gamma$ ray beam and the detector are aligned at opposite side with respect to the sample. The transmitted radiation has the same energy as the emitted one, but a lower intensity. When the sample is inhomogeneous then Eq. (1) may be written in the form:

$$NT = N_0 \exp (-\Sigma \mu_i \Delta x)$$

or

$$\Sigma \mu_i \Delta x = \ln \frac{N_0}{NT}$$

(2')

where $\mu_i$ is the constant attenuation coefficient in the i-th voxel. The reconstructed image is, therefore, representing a map of the linear attenuation coefficient, which, keeping constant the energy of incident radiation, depends of the mean atomic number $Z$, and on the physical density $\rho$.

For best images quality the condition $\Sigma \mu_i \Delta x \approx 2$ should be valid and in this hypothesis $NT \approx N_0 e^{-2} \approx 0.14 N_0$.

Generally the linear attenuation coefficient in the X-ray range depends on both photoelectric and Compton effect, plus an almost constant but small contribution due to Rayleigh scattering, which rarely exceeds 10% of the total attenuation coefficient [1].

At low energies, where the photoelectric effect is largely prevailing, the linear attenuation coefficient approximately depends on a high power of the mean atomic number ($\approx Z^3-Z^2$), while at higher energies, where the Compton effect is prevailing, it depends approximately on the physical density $\rho$ only. The terms low and high energy strongly depend on the atomic number of the material. For example for biological materials the region where photoelectric effect strongly prevails extends up to about 20 keV, for Al and Al-alloys up to about 40 keV;
Compton effect strongly prevails starts at about 60 keV for biological materials and at about 100 keV for Al.

Selective maps of single chemical elements can also be obtained, by using two monoenergetic radiation with energies bracketing the photoelectric discontinuity of this element [2]

2b. Compton tomography

In the case of Compton tomography, photons are detected which interact with electrons of the sample through Compton (or inelastic) scattering [1]. It is useful to put the detector at about 90° to better delimit the irradiated volume. With this geometry the access from both sides of the sample is not needed, and, therefore, also objects can be scanned (for example walls) that cannot be analyzed by transmission CT. The energy $E_C$ of 90° scattered photons is related to incident energy $E_0$ in the following manner:

$$E_C \approx \frac{E_0}{1 + \alpha}$$

where $\alpha = E_0/511$.

For $E_0 = 20, 40, 60, 80$ and $100$ keV, $E_C = 19.2, 37.1, 53.7, 69$ and $83.6$ keV. Therefore secondary Compton photons have an energy value not too different from that of the incident photons. The “Compton peak” has a mean energy value given by Eq. (3), but it is large, due to the angular spread, to multiple scattering and to the so called “Compton profile”.

Quantitatively, the number of $90°$ scattered photons by the irradiated volume can be easily calculated as:

$$N_C \approx N_0 \mu_C \times \Omega A$$

where:
- $\mu_C \propto \rho$ is the Compton attenuation coefficient at incident energy ($\rho$ represents the physical density);
- $\Omega$ is the solid angle sample – detector;
- $A$ is the attenuation of incident and scattered radiation.

In the hypothesis $\Sigma_i \mu_i \Delta x \approx 2$, then $A \approx e^{-2}$ as for transmission tomography, and putting the detector (radius of entrance window $r \approx R/2$) at a distance $2R$ ($R$ is the radius of the sample, supposed a cylinder), it may be deduced:

$$N_C \approx N_0 e^{-2} \left(\frac{1}{64}\right) \mu_C \times$$

Intensity of Compton photons is, therefore, about two orders of magnitude lower than that of transmission photons. As observed above it depends on the physical density of the irradiated voxel.

With the above described experimental conditions, full width at half maximum $\Delta E_C$ of the Compton peak, due to the scattering angle spread, is approximately given by: $\Delta E_C \approx E_0/2 \{1/(1+2\alpha)\}$.

2c. Elastic (Rayleigh and/or diffraction) scattering and tomography

In the case of elastic scattering, two effects should be considered: Rayleigh scattering, which corresponds to elastic scattering of photons by the bound electrons of a free atom, and Bragg effect, when the scattering due to different atoms gives rise to interference phenomena.

In the case of Rayleigh measurements and tomography, photons are employed which interact with the sample atoms through Rayleigh (or elastic) scattering. Photons are considered of energy $E_0$ scattered at low angles by the irradiated cylinder in the forward direction, because only in this conditions elastic scattering prevails over Compton scattering.

The intensity of forward scattered photons is approximately given by:

$$N_R \approx N_0 \mu_R \times \Omega A'$$

where $\mu_R$ is the Rayleigh and total attenuation coefficient at incident energy, $\Omega'$ is the sample-detector solid angle and $A'$ represents the attenuation of Rayleigh scattered radiation by the sample.
(\( A' \approx A \)). \( N_R \) is forward peaked and strongly depending on the scattering angle. Considering this fact and putting the detector (with radius \( R/2 \)) as close as possible to the sample (at a distance \( R \)) it may be deduced that: \( N_R \approx e^{2 \left( 1/16 \right)} \mu_R \times N_0 \) approximately.

In the best conditions, intensity of Rayleigh scattered photons is more than two orders of magnitude lower than that of transmitted photons. In the forward direction it depends on \( Z^2 - Z^3 \) of the irradiated volume.

For low-momentum transfer, i.e. low energy photons and/or small scattering angles (the momentum \( q \) is defined as \( q = \sin(\theta/2)/\lambda \)), solid-state scattering processes should be considered [3]. For \( q <\approx 1 \text{ Å}^{-1} \), diffraction peaks due to interatomic interaction can be detected, which depends on the crystal structure of the scattering material. A selective discrimination of a given crystal in the scanned specimen can be obtained by fixing the Bragg angle which produces an interference peak and, then, to carry out the computed tomography in the standard mode.

Rayleigh tomography should be especially suited to detect variations in the atomic number of the sample.

2d. X-ray fluorescence measurements and tomography

As observed in the Introduction, after a photoelectric effects secondary photons are emitted, which energy is characteristic of the elements of the sample [1]. These photons are X-rays, and can be employed to selectively image the presence of corresponding elements in the sample. However, only X-rays of sufficient energy will be useful, in a matrix of low atomic number elements and/or size, when the autoattenuation effects are reduced [4].

The intensity of secondary fluorescent X-rays for an element \( a \), emitted isotropically by the sample, is approximately given by:

\[
N_{ph,a} \approx N_0 \mu_{ph,a} \times \Omega'' \omega_a \AA''
\]

where \( \mu_{ph,a} \) represents the photoelectric attenuation coefficient for element \( a \) at incident energy \( E_0 \), \( \omega_a \) is the fluorescent coefficient, \( \Omega'' \) the solid angle sample-detector, \( \AA \) and \( \AA'' \) the attenuation of incident and fluorescent radiation of element \( a \) respectively.

Taking into consideration all these parameters and considering that the intensity of secondary fluorescent radiation strongly depends on the element to be analyzed, it may be approximately deduced that \( N_{ph,a} \approx e^{2 \left( 1/16 \right)} \mu_{ph,a} \times N_0 \times \AA'' \), putting the detector (with radius \( r=\frac{R}{2} \)) as close as possible to the sample (at a distance \( R \)) and considering a medium atomic number element.

X-ray fluorescence tomography is particularly suited to map chemical elements (with medium and/or high atomic number) in low atomic number matrices.

2e. Bremsstrahlung incident radiation

When the incident radiation is not monoenergetic, as for example from a X-ray tube, then some additional considerations should be made.

When for example radiation from a 50 kV X-ray tube is employed, having a maximum intensity at about 20 keV, and the sample is a cylinder of 2.5 cm water or plastic materials, (corresponding to the condition \( \mu x \approx 2 \)), then the transmitted radiation beam is hardened, no photons of energy below 15 keV are present, and the intensity maximum is shifted at about 30 keV (Figure 2). The Rayleigh scattered beam is similar to the transmitted one.

The beam of Compton scattered photons arriving to the detector is attenuated and further hardened due to the prevailing Compton effect at higher energies. Photons are present below about 20 keV, and the maximum intensity is at about 33 keV (Figure 2).

When Bremsstrahlung radiation of higher energy is employed, for example from a X-ray tube working at 80 kV, with a maximum intensity at about 35 keV, and the sample is now a cylinder of 6 cm diameter (for the condition \( \mu x \approx 2 \)), then the transmitted Rayleigh and Compton beam are
hardened and similar, and have a maximum intensity at about 45 keV. No photons are present below about 30 keV (Figure 2) [5].

3. EXPERIMENTAL SET-UP

The experimental set-up, showing the translation rotation mechanics and the detectors, is shown in Figure 3. It includes a X-ray source which is strongly collimated, to delimit a small irradiated cylinder. Four detectors or detection systems can be employed for the various types of tomographs, i.e.:

3a. transmission tomography
A single detector may be employed, or an array of detectors, or a small Image Intensifier. For transmission tomography a detector with good energy resolution is not mandatory, and therefore a very simple and inexpensive NaI(Tl) detector is employed, even if with a detector having better resolution a region of interest could be selected in the X-ray spectrum, reducing the background contribution. The detector is collimated in the same manner as the X-ray source, to strongly reduce the contribution of forward scattered photons and the background.

3b. Compton tomography
In the Compton region and at 90° scattering angle, Compton scattered photons are largely prevailing, and, therefore, a detector with a good energy resolution is not needed. To improve the number of Compton photons the solid angle sample-detector should be increased. For this reason a large area X-ray NaI(Tl) detector is employed, located at about 90° with respect to the incident beam. The detector is collimated to approximately collect only scattered photons from the whole irradiated cylinder.

Alternatively the detector can be strongly collimated to collect photons from a given voxel only. In this case no reconstruction procedure is needed, and the detector counts are directly proportional to the physical density in the voxel, except for the attenuation in the sample. This last procedure can be employed only when a very intense X-ray source is available.

3c. Rayleigh (or diffraction) tomography
The geometrical conditions for Rayleigh tomography is critical, because Rayleigh scattering is the prevailing effect only at very small angles (4°-10° approximately). An energy discrimination is not mandatory, but can be useful to exclude fluorescent peaks and background effects. A detector with relatively large area is also useful for improving the solid angle sample-detector. A CdZnTe detector was employed [6], collimated in order to collect only photons, scattered at small angles and with a reduced angular spread, by the irradiated cylinder.

When monoenergetic incident radiation is employed, then elastic scattered radiation can be selected by using a single channel analyser, improving the quality of the image.

3d. Fluorescent X-ray tomography
For X-ray fluorescent tomography a single detector is located at about 90° with respect to the incident beam.

The identification of secondary fluorescent X-rays requires a high energy resolution but also sufficiently large area detector, to improve the photons number. A second CdZnTe detector is, therefore, used. X-rays emitted by a single element or by more elements are selected by using a single channel or a multi channel analyser.

Alternatively the detector can be strongly collimated to collect photons from a given voxel only, as in the case of Compton tomography. This method can be employed for example when the sample has an “infinite thickness”. In this case no reconstruction procedure is needed, and the detector counts are proportional to the concentration of the element under study, except for the
autoattenuation in the sample. However this procedure can be employed only when a very intense X-ray source is available, as in the case of synchrotron radiation.

4. RECONSTRUCTION ALGORITHMS AND SELF ABSORPTION CORRECTIONS

Backprojection algorithms were used for transmission tomography, which are well known and described elsewhere[7]. However, these algorithms cannot be used for scattering and X-ray fluorescence tomography, because while transmission tomography uses the X-ray attenuation as information source, the attenuation of both primary and secondary radiation is not only useless for scattering or X-ray fluorescence tomography, but even represents a source of artifacts.

Thus, different classes of specific algorithms have been developed [8-10]. All of them consider, of course, the attenuation effects, but with different approaches. The simplest [8] represents a modification of the backprojection algorithm including the attenuation effects in the sample. It can be resumed with the following expressions:

\[ I(\theta, s) = \sum_{u=-\infty}^{\infty} I_0 f(\theta, s, u) p(s, u) g(\theta, s, u) \]  

(7)

where:
- \( I \) is the photon intensity at the detector;
- \( I_0 \) is the intensity from the X-Ray source;
- \( s, u \) and \( \theta \) are the coordinates of the rotational system;
- \( p(s, u) \) represents the characteristics of the interaction point (for example physical density for Compton tomography);
- \( f_0 \) and \( g_0 \) are the attenuation before and after the interaction point.

Eq. (7) uses a rotational system of coordinates. The data are then collected and backprojected and, finally, the scattering (or fluorescence) values are obtained:

\[ p(s_o, u_o) = \frac{\bar{I}_{\text{total}}(s_o, u_o)}{I_0 \sum_{i=1}^{n} f_0(\theta_i, s_i, u_i) g_0(\theta_i, s_i, u_i)} \]  

(8)

where \( \bar{I}_{\text{tot}}(\cdot, \cdot) \) is the contribution from all the projections at the point \((s_o, u_o)\) after a convolution with the Shepp-Logan filter.

Other algorithms use maximization/minimization techniques or modification of the algebraic recursive techniques ART. Their performance are in principle superior than the Hogan algorithm but the majority of published results were until now obtained for small matrix (and low resolution) or in condition of low attenuation. A new high performance algorithm was recently presented, which promise to give rise to real quantitative results [11].

5. RESULTS AND DISCUSSION

5a. Transmission and Compton X-ray tomography

As observed in the Introduction, transmission X-ray tomography has revolutionized the field of diagnostic Medicine and also, to a lower extent, the field of non destructive testing. Transmission Tomography is generally better than other kinds of X-ray images.

At the contrary, Compton tomography was never systematically explored and employed, and his potentiality is, at least partially, unknown.

Compton tomography has two important advantages:
-X-ray source and detector must not be aligned at the two sides of a sample, but both can be located at the same side; therefore, Compton tomography can be carried out also when the sample is not accessible on both sides; this point can be for example useful to examine walls (detachment in frescoes, detection and localization of leaks in internal pipes, and etc.) or partially analyse big samples not crossed by any X or γ-rays; the contrast for a small defect of size Δx in a matrix of attenuation coefficient μ is given by C=μΔx for transmission images, and C=Δμ/μ for Compton images (μ is the attenuation coefficient of the matrix, and Δμ indicates the difference between attenuation coefficient of the matrix and air respectively). In the case of Compton tomography the contrast is not proportional to the defect size, and can be better visualized [12]. Figure 4 shows the comparison between transmission and Compton tomography for a lucite test object. Transmission image has a better quality than the Compton one [13]. Figure 5 shows the comparison between transmission and Compton tomography for a cork [14]. In this case Compton image is much more detailed than the transmission image. It should be observed that the cork density is approximately 0.3-0.5 g/cm³, much lower than water density. Compton scattering measurements was also employed in a situation simulating the detachment of a fresco from the wall [15].

5b. Rayleigh and Diffraction X-ray tomography

Rayleigh tomography is related to forward photons elastically scattered at small angles. Working with monoenergetic incident radiation it should evidenciate the crystalline structure of matter, whereas working with bremsstrahlung radiation with a detector not too strongly collimated, the final image also strongly depends on the atomic number of the matrix. Only a few examples exist in the literature of Rayleigh images, and this area needs to be explored much more into details [16]. Figure 6 shows an example of Rayleigh tomography, for the same test object shown in Figure 4, and Figure 7 shows an interesting example of diffraction tomography, in which it may be observed that elements can be identified and selectively imaged, on the basis of their crystal structure [17]. Identification of explosives through selective diffraction radiography and tomography seems, for example, to be possible in the future [18,19].

5c. X-ray fluorescence tomography

X-ray fluorescence tomography is a very peculiar X-ray imaging technique, because it allows to map a single element distribution in the sample [20]. This technique is not of general use, because it can be employed only when X-rays of sufficient energy are emitted from the sample, able to cross the sample and to be detected. Therefore, a low-Z matrix is generally required, where medium-high atomic number elements should be mapped. Considering for example a water matrix sample of a few cm radius, elements from about Z=45 can be mapped, emitting X-rays with energy of 20 keV or more.

X-ray fluorescence tomography (and much more microtomography) is currently employed and gives very satisfactory results, especially when working with synchrotron radiation. However this technique was never combined with other tomographic methods. Figure 8 gives an example of XRF-tomography by using a X-ray tube [21].

6. CONCLUSIONS

From Sections 2a to 2d may be deduced that intensity of transmitted photons is approximately two orders of magnitude higher than that of Compton, Rayleigh and fluorescent X-ray tomography. That implies that a Compton or Rayleigh tomography takes much more time than a transmission tomography. A reduction of the transmitted photons intensity can be obtained for example by reducing the incident energy with respect to the condition μx ≈ 2, but that will not
produce a noticeable increase of scattered photons. The only parameters which can be used to increase the number of secondary photons are the geometrical factor and the probability for Compton, Rayleigh or fluorescence effects. The first can be improved by putting the detector as close as possible to the sample, as shown in Sections 2b to 2d, while the relative probability for Compton, Rayleigh and photoelectric cross section may be improved only by a proper selection of incident energy.

CT remains, of course, the most useful imaging technique using X-rays. As observed above, the limitations of this method are related to big samples that are not crossed by any X or γ radiation, or to samples not accessible from both sides or which cannot be moved with respect to the source-detector system. Further it suffers from the small contrast if the material under inspection are of similar atomic number and specific gravity.

Compton tomography can be also employed in the cases in which the samples are not accessible from both sides and, for imaging of parts of big samples. Compton tomography can also give better results in the case of voids and fractures in low density samples.

Rayleigh tomography was not sufficiently explored to deduce conclusions about its potentiality, while diffraction tomography can be extremely useful to selectively image crystal structures, and might be used as a kind of fingerprint or to detect contaminants in a given object, as in the case of detection of explosives in the baggage.

X-Ray Fluorescence Tomography can be useful for identification and mapping of medium-high atomic number elements in low-density matrices, and is especially suited for small samples, where the attenuation of fluorescent radiation is limited.

REFERENCES


Figure 1 – Schematic description of the interaction processes with matter of a beam of monoenergetic photons of energy $E_0$, for tomographic uses. The transmitted beam may be employed, as well as scattered (Compton and Rayleigh) and secondary X-ray fluorescence photons.
Figure 2 – Incident Bremsstrahlung radiation of 50 and 80 kV, and spectrum of transmitted and Compton scattered radiation. The partial monochromatization and energy shift of the spectra is shown. The condition $\mu x \approx 2$ is applied for the sample dimensions.
Figure 3 – Experimental set-up for the multi-tomography X-ray CT-scanner. The translation-rotation mechanics is shown, and three of the four employed detectors. A strongly-collimated small scintillation X-ray detector is employed for transmission tomography, a weakly collimated larger size scintillation detector for Compton tomography, and a shielded, not collimated CdZnTe detector for X-ray fluorescence analysis and tomography. A small, collimated second CdZnTe detector for Rayleigh (or diffraction) tomography was employed, but is not shown in the Figure.
Figure 4 – Comparison between transmission (left) and Compton CT – images for various test objects. From the top: a hollow lucite cylinder; the same cylinder with a graphite cylinder at the interior; a lucite box with nine holes; a walnut. NaI(Tl) scintillation detectors were employed both for transmission and Compton tomography.
Figure 5 – Compton tomographs of three cork samples of different quality, carried out at 40 kV, 1 mA. Photons intensity of the indicated profile is also shown in the right. The quality of the Compton image is better than transmission tomography image [14].
Figure 6 – Rayleigh tomography on the same hollow cylinder with graphite described in Figure 3.
Figure 7 – Diffraction images of a lucite cylinder of 8 mm diameter, containing two 1 mm diameter platinum and dysprosium wires. In the left the diffraction pattern of Pt and Dy are shown versus momentum q. In the right transmission tomography is shown (a) as well as diffraction images at the characteristic angle for Pt and Dy [17].
Figure 8 – X-Ray Fluorescence image of a LiF crystal (1 cm x 1 cm section) in the center of which a hole was made and a Silver solution poured and then extracted after some time. The diffusion of Ag-ions in the crystal is shown [20].